

Important Notice:

- ♣ The answer paper must be submitted before the deadline.
- ♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.
- ✂ Each answer paper must include your name and student ID.

1. Let X be a normed space and let E be a subspace of X . Put

$$\overline{E}^w = \{x \in X : \text{there is a sequence } (x_n) \text{ in } E \text{ so that } x_n \rightarrow x \text{ weakly}\}.$$

Show that \overline{E}^w is equal to the norm closure \overline{E} of E .

2. Let X and Y be normed spaces. Let $T : X \rightarrow Y$ be a bounded linear operator.

- (i) Show that $\ker T^* = \{y^* \in Y^* : y^* \circ T = 0\}$.
- (ii) Show that the image of T is dense in Y if and only if the adjoint operator T^* is injective.

*** **End** ***